C & EE 141

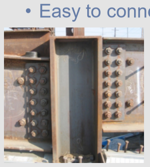
Bending Members:
Laterally Supported and Compact

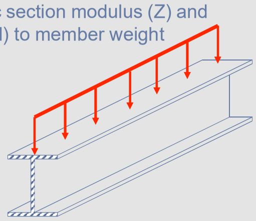
Flexural Members

- Structural members which support transverse loads, and are therefore subjected primarily to bending.
- For pure flexural members, axial load is negligible and ignored.
- If axial load is significant, it is a beam-column (we'll address that in later lectures).
- Also subjected to shear forces.
- Addressed in Part 3 of SCM and Section F of Spec

Wide Flange Beams

- Focus on Wide Flange shapes
  - Most common beam shape due to economy.
    - High ratio of plastic section modulus (Z) and moment of inertia (I) to member weight
    - Easy to connect



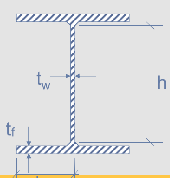


Laterally Supported Beams

- Compression flange (usually top) is restrained against global buckling due to compression component of bending stresses.
  - Metal deck puddle-welded to top flange of beam.
  - Headed studs engaging a concrete deck or slab.
  - Other details that prevent lateral movement of the flanges.

Compact Beams

- Similar to "non-slender" sections for compression
- Compact beams can develop yield stresses due to flexure without local buckling of any elements of the section.



$$h/t_w \le 3.76\sqrt{E/F_y}$$
(Spec Table B4.1b #15)

$$b/t \le 0.38\sqrt{E/F_y}$$
(Spec Table B4.1b #10)

Shapes with non-compact webs and/or flanges must consider local buckling limit states. (Much more complicated).

Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

- *Only limited to these three limit states when:*
  - *compression flange is laterally braced*
  - *beam is compact*

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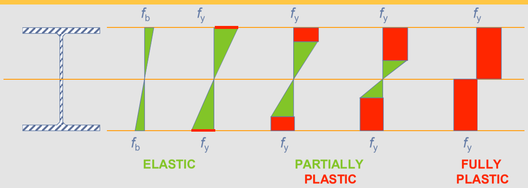
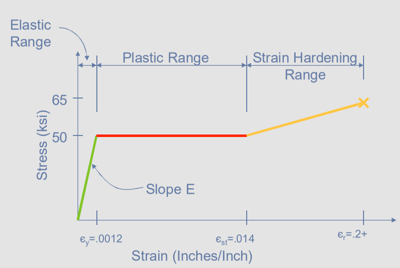
Flexural Stress and the Plastic Moment



As a beam receives increasing amounts of load, the stresses in the beam move from elastic to plastic. The plastic stresses overtake the elastic stresses for the entire depth of the shape. When this occurs, the beam has reached its plastic moment capacity.

Idealized Stress-Strain Curve for Grade 50 Steel

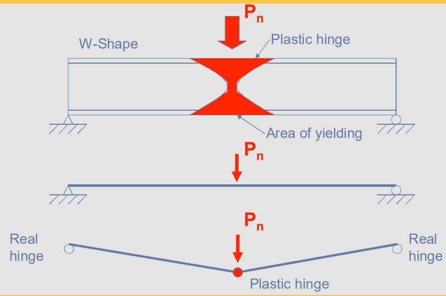


Plastic Moment

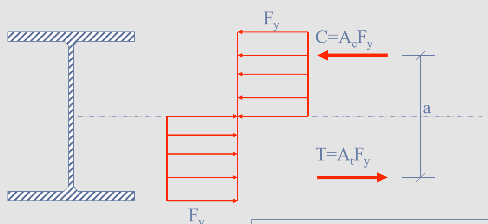
Actual Hinging

Idealize Loading

Idealized Hinging



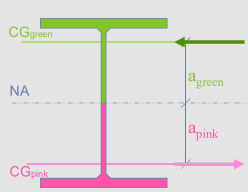
Calculating the Plastic Moment (M_p)



- M_p is the couple formed by T and C
- $M_p = F_y Z$ where:
  - Z = Plastic Section Modulus

Plastic Section Modulus (Z)

1. Calculate the neutral axis of the entire section
2. Calculate the centroid of each piece above and below the neutral axis
3. $M_p = F_y*(A_{green}*a_{green} + A_{pink}*a_{pink})$
4. $Z = A_{green}*a_{green} + A_{pink}*a_{pink}$
5. $M_p = F_y Z$



Nominal Plastic Flexural Capacity

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric I-shaped members and channels bent about their major axis, having compact webs and compact flanges as defined in Section B4.1 for flexure.

> **User Note:** All current ASTM A6 W, S, M, C and MC shapes except W21×48, W14×99, W14×90, W12×65, W10×12, W8×31, W8×10, W6×15, W6×9, W6×8.5 and M4×6 have compact flanges for F_y = 50 ksi (345 MPa); all current ASTM A6 W, S, M, HP, C and MC shapes have compact webs at $F_y \le 65$ ksi (450 MPa).

The *nominal flexural strength*, M_n, shall be the lower value obtained according to the *limit states* of *yielding (plastic moment)* ~~*and lateral-torsional buckling*~~.

(LTB does not apply with continuous bracing)

Nominal Plastic Flexural Capacity

1. Yielding

$$M_n = M_p = F_y Z_x \qquad \text{(F2-1)}$$

where

F_y = *specified minimum yield stress* of the type of steel being used, ksi (MPa)

Z_x = plastic section modulus about the *x*-axis, in.^{3} (mm^3)

Spec F2

Basic Design Equations for Beams

- $M_u \le \Phi_b M_n$

Where:

- M_u = Required Moment Strength
- M_n = Nominal Moment Strength
- Φ_b = 0.9

Do We Need All These Equations?

- Refer to AISC SCM
  - Section Properties
    - Table 1-1: lists S, I, Z
  - Beam Tables
    - Table 3-2: select most economical size based on Z_x
    - Table 3-3: select most economical size based on I_x

Beam Tables Based on Z_x

- Strong Axis Plastic Section Modulus
- Very Commonly Used Table

Table 3-2 (continued)
W-Shapes
Selection by Z_x

F_y = 50 ksi

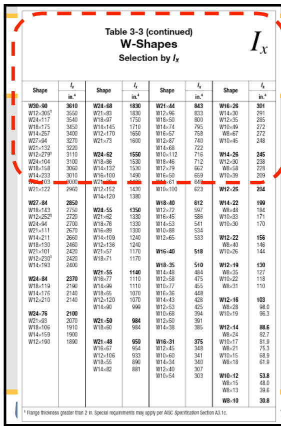
Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W30×116	**378**	**943**	**1420**	**575**	**864**	**24.8**	**37.4**	**7.74**	**22.6**	**4930**	**339**	**509**
W21×147	373	931	1400	575	864	13.7	20.7	10.4	36.3	3630	318	477
W24×131	370	923	1390	575	864	16.3	24.6	10.5	31.9	4020	296	445
W18×158	356	888	1340	541	814	10.5	15.9	9.68	42.8	3060	319	479
W14×193	355	886	1330	541	814	5.30	7.93	14.3	79.4	2400	276	414
W12×210	348	868	1310	510	767	4.25	6.45	11.6	95.8	2140	347	520
W30×108	**346**	**863**	**1300**	**522**	**785**	**23.5**	**35.5**	**7.59**	**22.1**	**4470**	**325**	**487**
W27×114	343	856	1290	522	785	21.7	32.8	7.70	23.1	4080	311	467
W21×132	333	831	1250	515	774	13.2	19.9	10.3	34.2	3220	283	425
W24×117	327	816	1230	508	764	15.4	23.3	10.4	30.4	3540	267	401
W18×143	322	803	1210	493	740	10.3	15.7	9.61	39.6	2750	285	427
W14×176	320	798	1200	491	738	5.20	7.83	14.2	73.2	2140	252	378

ASD	LRFD
$\Omega_b = 1.67$	$\phi_b = 0.90$
$\Omega_v = 1.50$	$\phi_v = 1.00$

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 50 ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

Beam Tables Based on I_x



- Strong Axis Moment of Inertia
- Very Commonly Used Table

Table 3-3 (continued)
W-Shapes
Selection by I_x

I_x

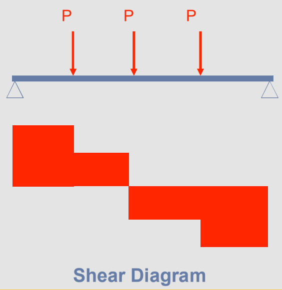
Shape	I_x in.^{4}	Shape	I_x in.^{4}	Shape	I_x in.^{4}	Shape	I_x in.^{4}
W30×90	**3610**	**W24×68**	**1830**	**W21×44**	**843**	**W16×26**	**301**
W12×305[h]	3550	W21×83	1830	W12×96	833	W14×30	291
W24×117	3540	W18×97	1750	W18×50	800	W12×35	285
W18×175	3450	W14×145	1710	W14×74	795	W10×49	272
W14×257	3400	W12×170	1650	W16×57	758	W8×67	272
W27×94	3270	W21×73	1600	W12×87	740	W10×45	248
W21×132	3220			W14×68	722		
W12×279[h]	3110	**W24×62**	**1550**	W10×112	716	**W14×26**	**245**
W24×104	3100	W18×86	1530	W18×46	712	W12×30	238
W18×158	3060	W14×132	1530	W12×79	662	W8×58	228
W14×233	3010	W16×100	1490	W16×50	659	W10×39	209
W24×103	3000	W21×68	1480	W14×61	640		
W21×122	2960	W12×152	1430	W10×100	623	**W12×26**	**204**
		W14×120	1380				

Limit States for Flexure

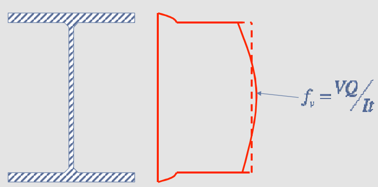
- Plastic Flexural Capacity
- Shear Capacity
- Deflection

- *Only limited to these three limit states when:*
  - *compression flange is laterally braced*
  - *beam is compact*

Calculation of Shear Stress



Examine Stress in a X-Sect



Clearly, f_v in the web governs the design over that in the flange. Also note that the average f_v is almost equal to the maximum f_v.

Therefore, we can re-define f_v in an easier way…

Basic Shear Strength Relationships

- Spec Chapter G: Design of Members for Shear
  - $V_u \leq \Phi_v V_n$ where:
    - V_u = Required shear strength
    - Φ_v = Reduction factor for shear
    - V_n = Nominal shear strength
- General case:
  - $V_n = 0.6 F_y A_w C_v$ (Eq G2-1)
  - $\Phi_v = 0.90$
  - C_v accounts for the web slenderness

Shear Limit States

- There are different shear limit states depending on the mode of shear failure
  - Shear yielding
  - Web buckling
- These limit states are dependent on the slenderness ratio of the web, h/t_w.
  - The value of C_v in Eq G2-1 varies depending on the value of h/t_w.
  - Generally, more slender web → smaller C_v

Typical W-Shape Shear Strength

- However, for webs of rolled I-shape members with

$$h/t_w \le 2.24\sqrt{E/F_y}$$

  - $\Phi_v = 1.00$
  - $C_v = 1.0$
- This applies to almost every rolled steel shape

User Note: All current ASTM A6 W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 meet the criteria stated in Section G2.1(a) for $F_y = 50$ ksi (345 MPa).

- See Spec G2.1(a)

AISC: Shear Strength

(b) For webs of all other doubly symmetric shapes and singly symmetric shapes and channels, except round *HSS*, the web shear coefficient, C_v, is determined as follows:

(i) When $h/t_w \le 1.10\sqrt{k_v E/F_y}$

$$C_v = 1.0 \quad \text{(G2-3)}$$

(ii) When $1.10\sqrt{k_v E/F_y} < h/t_w \le 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \quad \text{(G2-4)}$$

(iii) When $h/t_w > 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \quad \text{(G2-5)}$$

Spec G2.1

AISC: Shear Strength

The web plate *shear buckling* coefficient, k_v, is determined as follows:

(i) For webs without *transverse stiffeners* and with $h/t_w < 260$:

$$k_v = 5$$

except for the stem of tee shapes where $k_v = 1.2$.

(ii) For webs with transverse stiffeners:

$$k_v = 5 + \frac{5}{(a/h)^2} \quad \text{(G2-6)}$$

$$= 5 \text{ when } a/h > 3.0 \text{ or } a/h > \left[\frac{260}{(h/t_w)}\right]^2$$

where

a = clear distance between transverse stiffeners, in. (mm)

User Note: For all ASTM A6 W, S, M and HP shapes except M12.5×12.4, M12.5×11.6, M12×11.8, M12×10.8, M12×10, M10×8 and M10×7.5, when F_y = 50 ksi (345 MPa), C_v = 1.0.

Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

- *Only limited to these three limit states when:*
  - *compression flange is laterally braced*
  - *beam is compact*

Serviceability

- Serviceability is related to performance.
- A design must not only have sufficient strength, but also must not cause significant discomfort for users.
- Deflections and vibrations are the most common serviceability considerations.
- As such, we impose limits on service load deflections which will provide good serviceability.

Deflection Criteria

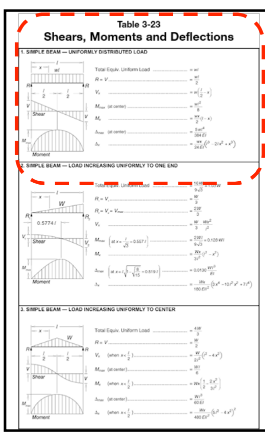
- Typical Criteria:
  - Live Load Deflection Held to Less Than L/360
  - Dead + Live Load Deflection Held to Less than L/240.
  - L is Measured in Inches.
  - Loads Are Service Loads!

LOADS ARE SERVICE LOADS!

Summary of Beam Design with Lateral Support

1. Establish Loading (Assuming self-weight)
2. Find M_u
3. Use $Z_{x\ rqd.} = M_u/\Phi_b F_y$; Use Table 3-2 to select a section with required Z_x.
4. Check assumption of beam weight; Iterate as required.
5. Check Shear; Iterate as required.
6. Check Deflection; Iterate as required.

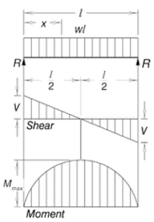
Tools for Calculating Internal Forces & Deflections



- Most loading conditions encountered in design are solved in the manual.
- HUGE TIME SAVER

Table 3-23
Shears, Moments and Deflections

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) .... $= \frac{wl^2}{8}$

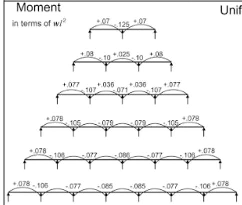
M_x .... $= \frac{wx}{2}(l - x)$

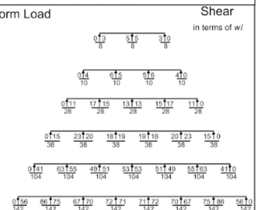
Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

Table 3-22c
Continuous Beams
Moments and Shear Coefficients—
Equal Spans, Equally Loaded

Moment in terms of wl^2 | Uniform Load | Shear in terms of wl





Questions?

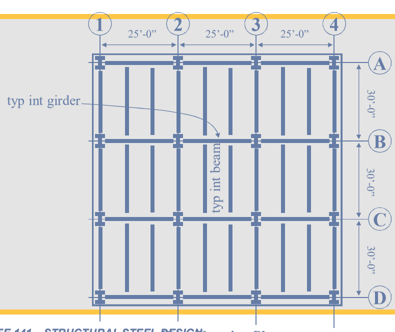
UCLA CEE 141 – STRUCTURAL STEEL DESIGN

Example Problem

- Design the most efficient typical interior WF beam.
- Design the most efficient typical interior WF girder.

- Assumptions:
  - Floor Plans are on the following page
  - Floor Loading: Dead = 100psf, Live = 40 psf (reducible)
  - Self weight of the members is included in the dead load
  - A992 Steel
  - Beam compression flanges are fully braced by headed studs to concrete fill on metal deck
  - Deflection Limits: Dead + Live = L/240, Live = L/360
  - Live Load Reducible per ASCE 7-10

Example Problem



Floor Framing Plan

Example Problem

- **Typical Interior Beam**

  - Calculate Loads

    DL = 100 psf; LL_0 = 40 psf

    Trib Area = (25'/3) (30') = 250 ft^2

    $LL = LL_0\,[0.25 + 15 / \sqrt{K_{LL}\,A_T}]$

    $= (40psf)\,[0.25 + 15 / \sqrt{2 \times 250ft^2}]$

    = 37 psf

Example Problem

- Calculate Applied Loads
  - w_s = DL + LL = (100psf + 37psf) (25'/3) = 1.14klf
  - w_L = LL = (37psf) (25'/3) = 0.31klf
  - w_u = 1.2DL + 1.6LL = [(1.2) (100psf) + (1.6) (37psf)] (25'/3) = 1.49 klf
- Calculate Internal Forces

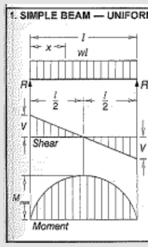
  $M_u = w_u L^2 / 8 = (1.49\text{klf}) (30')^2 / 8 = 168\text{k-ft}$

  $V_u = w_u L / 2 = (1.49\text{klf}) (30') / 2 = 22.4\text{k}$

Example Problem

- Table 3-23 (pg 3-211)

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) .... $= \frac{wl^2}{8}$

M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

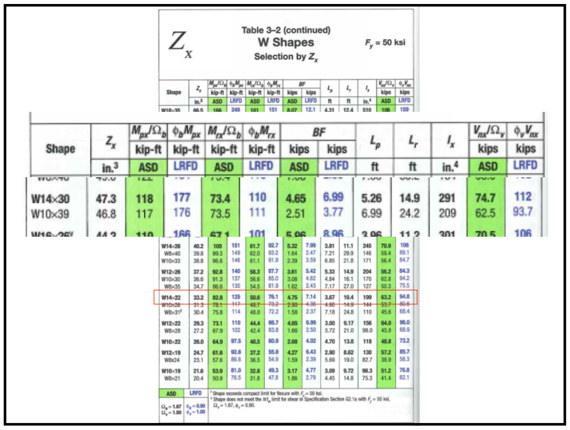
Example Problem

- Check Flexural Capacity

  Top Flange Fully Braced (L_b = 0')

  Z_{reqd} = $M_u / \phi_b F_y$ = (168k-ft) (12"/ft) / (0.9) (50 ksi) = 44.8 in^3
- ***Try W14x30*** (Z = 47.3 in^3)
- Check Shear Capacity

  $\phi_v V_n$ = $\phi_v (0.6) A_w F_y C_v$ = (1.0) (0.6) (13.84") (0.27") (50ksi) (1.0) = 121k *therefore OK for shear*



Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF	BF	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
	in.^{3}	kip-ft ASD	kip-ft LRFD	kip-ft ASD	kip-ft LRFD	kips ASD	kips LRFD	ft	ft	in.^{4}	kips ASD	kips LRFD
W14×30	47.3	118	177	73.4	110	4.65	6.99	5.26	14.9	291	74.7	112
W10×39	46.8	117	176	73.5	111	2.51	3.77	6.99	24.2	209	62.5	93.7

Example Problem

- Check Serviceability
  - Live Load Deflection

$$\Delta_{max} \text{ (at center) } \ldots\ldots = \frac{5wl^4}{384EI}$$

Δ_L = 5 w L^4 / 384 E I

= (5)(0.31klf)(30')^4(12"/ft)^3/(384)(29E3ksi) (291in^4)

= 0.67" < L/360 = (30')(12"/ft)/360 = 1" *therefore OK*

  - Dead Load + Live Load Deflection

Δ_{D+L} = 5 w L^4 / 384 E I

= (5)(1.14klf)(30')^4(12"/ft)^3/(384)(29E3ksi) (291in^4)

= 2.46" < L/240 = (30')(12"/ft)/240 = 1.5" *therefore NG*

Example Problem

- Check Serviceability
  - Determine I_{reqd}

I_{reqd} = [(2.46")/(1.5")] (291 in^4) = 477in^4

Options:

W18x35 (I = 510; Z = 66.5) *select lightest*

W16x40 (I = 518; Z = 72.9)

Use W18x35 typical interior beam

Example Problem

- **Typical Interior Girder**
  - **Calculate Loads**

    DL = 100 psf; LL_0 = 40 psf

    Trib Area = (2/3) (25') (30') = 500 ft^2

    $LL = LL_0\,[0.25 + 15 / \text{sqrt}(K_{LL}\,A_T)]$

    $= (40\text{psf})\,[0.25 + 15 / \text{sqrt}\,(2 \times 500\text{ft}^2)]$

    = 29 psf

Example Problem

- Calculate Applied Loads
  - P_D = (100psf) (25'/3) (30') = 25.0k
  - P_L = (29psf) (25'/3) (30') = 7.3k
  - P_s = $P_D + P_L$ = 25.0k + 7.3k = 32.3k
  - P_u = $1.2P_D + 1.6P_L$

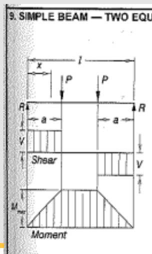
    = (1.2)(25.0k) + (1.6)(7.3k) = 41.7k
- Calculate Internal Forces

  $M_u = P_u\,L / 3$ = (41.7k) (25'/3) / 3 = 347kft

  $V_u = P_u$ = 41.7k

Example Problem

- Table 3-23 (pg 3-213)



9. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

Total Equiv. Uniform Load	$= \frac{8Pa}{l}$
$R = V$	$= P$
M_{max} (between loads)	$= Pa$
M_x (when $x < a$)	$= Px$
Δ_{max} (at center)	$= \frac{Pa}{24EI}(3l^2 - 4a^2)$
Δ_{max} (when $a = \frac{l}{3}$)	$= \frac{Pl^3}{28EI}$
Δ_x (when $x < a$)	$= \frac{Px}{6EI}(3la - 3a^2 - x^2)$
Δ_x (when $a < x < (l - a)$)	$= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

Example Problem

- Check Flexural Capacity
  - Top Flange Fully Braced (L_b = 0')
  - Z_{reqd} = M_u/ $\phi_b F_y$
    = (347k-ft) (12"/ft) / (0.9) (50 ksi)
    = 92.6 in^3
- ***Try W21x44*** (Z = 95.4 in^3; I = 843 in^4)
- Check Shear Capacity
  - $\phi_v V_n$ = 195k *from tables*
    > V_u = 41.7k

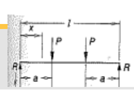
F_y = 50 ksi

Table 3-2 (continued)
W Shapes
Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF	BF	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
	in.^{3}	kip-ft	kip-ft	kip-ft	kip-ft	kips	kips	ft	ft	in.^{4}	kips	kips
		ASD	LRFD	ASD	LRFD	ASD	LRFD				ASD	LRFD
W21×44	95.4	238	358	143	214	11.2	16.8	4.45	13.0	843	145	217
W16×50	92.0	230	345	141	213	7.59	11.4	5.62	17.2	659	124	185
W18×46	90.7	226	340	138	207	9.71	14.6	4.56	13.7	712	130	195
W14×53	87.1	217	327	136	204	5.27	7.93	6.78	22.2	541	103	155
W12×58	86.4	216	324	136	205	3.76	5.66	8.87	29.9	475	87.8	132
W10×68	85.3	213	320	132	199	2.57	3.86	9.15	40.6	394	97.8	147

Example Problem



- Check Serviceability
  - Live Load Deflection
    - Δ_L = (P a / 24 E I) (3l^2 – 4a^2) Δ_{max} (at center) .... $= \frac{Pa}{24EI}(3l^2 - 4a^2)$
      = [(7.3k)(25'/3)(12"/ft)^3/(24)(29E3ksi)(843in^4)] [3(25')^2-4(25'/3)^2]
      = 0.29" < L/360 = (25')(12"/ft)/360 = 0.83" *OK*
  - Dead Load + Live Load Deflection
    - Δ_{D+L} = *ratio the deflections*
      = (32.2k/7.3k) (0.29")
      = 1.27" ~ L/240 = (25')(12"/ft)/240 = 1.25" *close OK*

Use W21x44 typical interior girder